# **Dynamics of Small Open Economies**

Econ 4330 Open Economy Macroeconomics Spring 2008

Lecture 4 Part A

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$$CA_t = Y_t - \tilde{Y}_t - (I_t - \tilde{I}_t) - (G_t - \tilde{G}_t) + \frac{v}{1+r}W_t$$

## Lessons about CA from the infinite horizon models

- Temporarily high output causes surpluses
- Temporarily high investment or gov. consumption cause deficits
- Temporarily high or low taxes do not matter.
- The more persistent shocks are, the lower is the initial CA effect
- Expectations of fast-output growth produce deficits
- A high degree of impatience among consumers produce deficits
- Countries with a high marginal productivity of capital tend to get an initial deficit if they open up to international capital markets
- Deficits or surpluses can continue indefinitely (do not self-correct)

Do the conclusions fit with experience?

Do the conclusions survive if we enrich the theory?

# The effect of output growth on CA

Consumption growth determined by Euler equation:  $u'(C_t) = \beta(1+r)u'(C_{t+1})$ Output growth determined by exogenous productivity growth (and investment) Local consumption growth is independent of local output growth

Euler equation with CES-utility:  $C_{t+1} = [\beta(1+r)]^{\sigma}C_t = (1+v)C_t$  v = rate of growth of consumption If  $\beta < 1$  and  $\sigma < 1$ , v < r. Assume this.

Compare two countries with different rates of output growth

# Solution of the small open economy model

From last weeks lecture with  $I_t = 0$  and  $G_t = 0$ 

$$W_t = (1+r)B_t + \frac{1+r}{r}\tilde{Y}_t$$

$$C_t = \frac{r-v}{1+r}W_t = (r-v)B_t + \frac{r-v}{r}\tilde{Y}_t$$

$$CA_t = rB_t + Y_t - C_t = Y_t - \frac{r-v}{r}\tilde{Y}_t + vB_t$$

# Consequences of different rates of trend output growth

$$Y_{t+1} = (1+g)Y_t$$

g = outupt growth rate, r > g assumed

$$\tilde{Y}_{t} = \frac{r}{1+r} \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (1+g)^{s-t} Y_{t} = \left(\frac{r}{1+r}\right) \left(\frac{1}{1-\frac{1+g}{1+r}}\right) Y_{t} = \frac{r}{r-g} Y_{t}$$

Insert this in the current account equation:

$$CA_t = Y_t - \frac{r - v}{r} \left(\frac{r}{r - g}\right) Y_t + vB_t = \frac{v - g}{r - g} Y_t + vB_t$$

(v-g)/(r-g) is the savings rate out of current income from production

From interest income  $vB_t$  is saved, while  $(r - v)B_t$  is spent

Stricter rule than for the Petroleum Fund!

 $v>0 \Rightarrow$ ,  $B_t>0$  contributes to  $CA_t>0$ , and increased  $B_{t+1}$ . No self-correction.

$$CA_t = \frac{v - g}{r - g}Y_t + vB_t$$

## The country with low output growth

g < v Output growth lower than consumption growth

- The share of output saved (v-g)/(r-g), is positive.
- The share can be huge even if *v-g* is small
- The savings are needed to raise future consumption possibilities faster than income.
- The staring level of consumption is low

## The country with high output growth

g > v: Output growth higher than consumption growth

- The share of output saved (v-g)/(r-g), is negative.
- The starting level of consumption is high.

# What happens to the debt in the long run?

$$B_{t+1} = B_t + CA_t = (1+v)B_t + \frac{v-g}{r-g}Y_t$$

Asset ratio:  $b_t = B_t/Y_t$  (negative of the debt ratio)

$$\frac{B_{t+1}(1+g)}{Y_t(1+g)} = (1+v)\frac{B_t}{Y_t} + \frac{v-g}{r-g}$$

$$b_{t+1}(1+g) = (1+v)b_t + \frac{v-g}{r-g}$$

$$b_{t+1} = \frac{1+v}{1+g}b_t + \frac{v-g}{(1+g)(r-g)}$$

First order difference equation, solution see Berck and Sydsæter

$$b_{s} = \left(\frac{1+v}{1+g}\right)^{s-t} \left(b_{t} + \frac{1}{r-g}\right) - \frac{1}{r-g}$$

$$b_{s} = \left(\frac{1+v}{1+g}\right)^{s-t} \left(b_{t} + \frac{1}{r-g}\right) - \frac{1}{r-g}$$

## *High output growth country g > v*

Then 
$$s \to \infty \Rightarrow b_s \to -\frac{1}{r-g}$$

- Debt ratio goes to a (high) constant
- High output growth → consumption exceeds output to begin with.
- The share of consumption in output will tend to zero.
- The whole GDP is used to pay interest on the foreign debt

## Low output growth country g < v

- $b_t > -1/(r-g)$ , has to be positive initially (or country is bankrupt)
- $b_s$  will become positive and grow without limit.
- Some of the interest is still consumed.

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# The long run solution is not meaningful

If the consumption growth rate exceeds the output growth rate forever, the share of consumption in output will tend to zero.

Sooner or later the small country ceases to be small

Default risks

Is the present value budget constraint too permissive?

Constraints on the debt to GDP-ratio will force fast-growing countries to borrow less

Constant growth rates forever? No!

# Are consumers really behaving as if they have an infinite horizon?

Bequests are common

Children do support their parents

Maximizing a dynastic welfare function?

Following social norms or biological urges?

Why do poor parents leave bequests to rich descendants?

Bequests determined by the parents' resources?

Or by the difference between the parent and the children?

# Alternative theories of saving

## Life-cycle saving:

- Saving for old age, college for the children, dowry, (bequests)
- Saving and accumulated wealth related to life-time income of present generations

## **Precautionary savings**

- Risks that cannot be insured
- Risks not covered by social security
- Desired wealth stands in relation to income
- Credit rationing

#### Power and status

- Business, saving profits to expand the business
- Politics

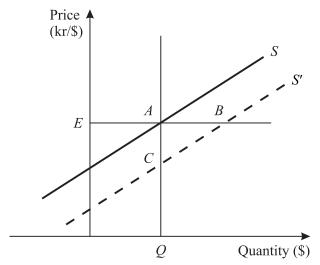
# The foreign exchange market ECON4330 Lecture 4A

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6th February 2008

## Equilibrium with fixed and floating exchange rates



Stock approach versus flow approach

#### Financial balance sheet

Assets	Govern.	Private	Foreign	Total
Kroner	$B_{g}$	$B_p$	$B_*$	0
Dollars	$F_{g}$	$F_p$	$F_*$	0
Total	$B_g + EF_g$	$B_p + EF_p$	$B_* + EF_*$	0

 $B_i$  = Net kroner assets of sector i

 $F_i$  Net dollar assets of sector i

 $F_{\rm g}=$  Foreign exchange reserves - Government foreign currency debt

$$B_i + EF_i = B_{i0} + EF_{i0}$$

Reallocation of a given portfolio within a short period

#### Factors determining portfolio choice

#### Rates of return:

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i on domestic currency
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 $i_* + e$  on foreign currency:

 $e = \dot{E}/E$  rate of depreciation, uncertain

Exchange rate expectations  $e_e$  (may differ between individuals) Exchange rate risk and risk aversion Capital controls, transaction costs Liquidity needs

## a)Perfect capital mobility

Investors care only about expected return Risk neutrality, or no exchange rate risk All have the same expected rate of depreciation,  $e_{\rm e}$ 

#### Only equilibrium:

$$i = i + e_e$$

Uncovered interest parity UIP

Investors are indifferent between B and F when UIP holds

## b)Portfolio choice with imperfect capital mobility

Define the risk premium on kroner:  $r = i - i_* - e_e$ 

$$W_{p} = \frac{B_{p0} + EF_{p0}}{P} = \frac{B_{p} + EF_{p}}{P}$$

$$\frac{EF_{p}}{P} = f(r, W_{p})$$

$$\frac{B_{p}}{P} = W_{p} - f(r, W_{p})$$

 $f_r < 0$  Higher risk premium on the domestic currency  $\Longrightarrow$  Portfolio shift from foreign to domestic currency  $0 < f_W < 1$  An increases in wealth will be invested in both currencies.

Diversification

#### Portfolio choice of the foreign sector

$$W_{*} = \frac{B_{*0}/E + F_{*0}}{P_{*}} = \frac{B_{*}/E + F_{*}}{P_{*}}$$

$$\frac{B_{*}}{EP_{*}} = b(r, W_{*})$$

$$\frac{F_{*}}{P_{*}} = W_{*} - b(r, W_{*})$$

 $b_r > 0.0 < b_W < 1$ 

#### The supply of foreign currency

$$F_g = -F_p - F_*$$

$$F_g = -\frac{P}{E}f(r, W_p) - P_*[W_* - b(r, W_*)]$$

$$F^{S}(E, i - i_{*}) = -\frac{P}{E}f(i - i_{*} - e_{e}(E), \frac{B_{p0} + EF_{p0}}{P})$$
$$-P_{*}[\frac{B_{*0}/E + F_{*0}}{P_{*}} - b(i - i_{*} - e_{e}(E), \frac{B_{*0}/E + F_{*0}}{P_{*}})]$$

E has 1) A portfolio composition effect and 2) An expectations effect Equilibrium condition determines  $F_g$  under fixed rates, E under floating.

## 1) The portfolio composition effect

$$E \uparrow \Longrightarrow \frac{EF_{\rho 0}}{P} \uparrow$$
 Wealth goes up

Investors want to keep only a fraction  $f_W$  of the capital gain in foreign currency.

The remainder  $1 - f_W$  is invested in domestic currency.

Investors sell an amount of foreign currency equal to  $1-f_W$  times the capital gain.

Supply of foreign currency to CB goes up.

#### Assumptions made:

 $F_{p0} > 0$  There is a capital gain.

 $f_W < 1$  Not all capital gains are invested in foreign currency.

Rebalancing of the portfolio

#### 2. The expectations effect

Assume regressive expectations  $e'_e < 0$  Example:

$$e_e = \alpha \frac{E_e - E}{E}$$
  $\alpha > 0$ 

 $E_e$  expected future exchange rate, constant  $\alpha$  speed of convergence

$$E \uparrow \Longrightarrow e_e \downarrow \Longrightarrow r \uparrow$$

Lower expected return on foreign currency

Sell foreign currency, buy domestic currency ( $f_r < 0$ )

Supply of foreign currency to CB goes up

#### The slope of the supply curve

$$\frac{\partial F^{S}}{\partial E} = \frac{P}{E^{2}} \gamma - \frac{P}{E} \kappa e'_{e}$$

where

$$\gamma = (1 - f_W) \frac{EF_{p0}}{P} + (1 - b_W) \frac{B_{*0}}{P}$$

and

$$\kappa = -f_r + \frac{EP_*}{P}b_r > 0$$

 $\kappa$  measures the degree of capital mobility between the two currencies  $\gamma$  measures the product of the exposure that each country has to the other currency and the share of capital gains that they will bring home.

Sufficient conditions for a positive slope are:

$$F_{p0} > 0, \ B_{*0} > 0, \ f_W < 1, \ b_W < 1, \ e'_e < 0$$

## Floating: The effect of i on E

Equilibrium condition:

$$F_g = F^S(E, i - i_*)$$

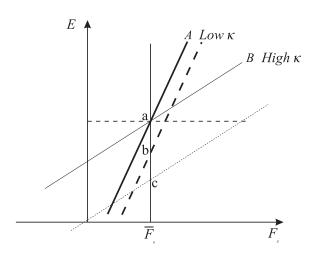
Differentiate:

$$dF_g = \frac{\partial F^S}{\partial E} dE + \frac{\partial F^S}{\partial i} di = 0$$

or

$$\frac{dE}{di} = -\frac{\frac{\partial F^{S}}{\partial i}}{\frac{\partial F^{S}}{\partial E}} = -\frac{\kappa}{\gamma/E - \kappa e'_{e}} < 0$$

$$\kappa \to \infty \Longrightarrow \frac{dE}{di} \to \frac{1}{e'_e}$$



High capital mobility increases the effect of i on E. The effect need not be great though  $i=i_*+e_e(E)$  and  $e_e(E)=\alpha\frac{E_e-E}{E}$  implies:

$$E = \frac{E_{\rm e}}{\alpha + i - i_*}$$

Expectations are crucial when capital mobility is high

#### Floating: The effect of an intervention

$$F_g = F^S(E, i - i_*)$$
$$dF_g = \frac{\partial F^S}{\partial E} dE$$

$$\frac{dE}{dF_g} = \frac{1}{\frac{\partial F^S}{\partial F}} = \frac{1}{(P/E^2)\gamma - (P/E)\kappa e_e'} > 0$$

CB buys foreign currency, the price of foreign currency goes up



CB sells domestic currency, domestic currency depreciates

$$\kappa \to \infty \Longrightarrow \frac{dE}{dF_g} \to 0$$

Signalling effects

## Fixed rate: The cost of keeping $i \neq i_*$

Assume  $e_e = 0$  (credible fix) CB's balance sheet:

$$B_g + EF_g = B_{g0} + EF_{g0}$$

High capital mobility

 $i >> i_* \Longrightarrow F_g$  large positive,  $B_g$  large negative.

 $i << i_* \Longrightarrow F_g$  large negative,  $B_g$  large positive.

CB borrows at high rate, lends at low rate in both cases

Large deviations from interest rate parity in both directions have a cost

Reserves may run out if i is set too low

 $\kappa \to \infty$  The cost of even small deviations from  $i = i_*$  becomes enormous

High confidence in a fixed rate

- → Little exchange rate risk
- $ightarrow e_e = 0$  and  $\kappa$  large
- $\rightarrow$  Must have  $i \approx i_*$

## Defence of a fixed rate against $e_e \neq 0$

Assume  $e_e > 0$  Devaluation expected

 $e_e \uparrow, \kappa$  large  $\rightarrow$  large loss of reserves Loss of reserves can be prevented if r is kept constant  $di = de_e$ Problem: Extremely high short term rates may be required

Troblem. Extremely high short term rates may be rec

Usual strategy:

$$i_* < i < i_* + e_e, \qquad dF_g < 0, \quad dB > 0$$
 CB profits if  $e < i - i_*$   
Speculators profit if  $e > i - i_*$ 

#### Defence difficult because

Reserves may run out.

Defence with extremely high i is not credible. Fear of debt crisis.

Fear of losses if CB must give in.

#### $e_e < 0$ Revaluation expected

Reserves cannot run out Impossible to have negative *i* 

#### Perfect capital mobility:

Interventions ineffective

Interest rate the only instrument which can keep the exchange rate fixed

#### Responses:

Mutual fixing Currency boards Wide margins

#### The current account

$$F_g = -\frac{P}{E}f(r, W_p) - P_*[W_* - b(r, W_*)]$$

Assume P,  $P_*$ , E, i,  $i_*$  constant over time

$$\dot{F}_g = -\frac{P}{E} f_W \dot{W}_p - P_* (1 - b_W) \dot{W}_* \tag{1}$$

$$W_p + W_g + \frac{EP_*}{P}W_* = 0$$
  
 $\dot{W}_p = -\dot{W}_g - \frac{EP_*}{P}\dot{W}_*$ 

Inserted in (1):

$$\dot{F}_g = \frac{P}{E} [1 - f_W - b_W] (-\dot{W}_*) + \frac{P}{E} f_W \dot{W}_g$$
 (2)

Condition for a current account surplus to increase reserves: